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PLASTIC DEFORMATION UNDER A GENERALIZED PROPORTIONAL LOADING

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Plastic deformation is mainly the result of the displacement of one part of a crystal with respect to another. This latter specified the creation of physical theories of residual deformations within the framework of the slip concept [1]. On the basis of one such model, an attempt is made in [2, 3] to set up a connection between the stress and strain in time. To do this, a temperature–time operator was introduced into the governing relationships. The operator is introduced from the following physical considerations.

As is known, plastic flow in a material is developed extremely inhomogeneously and results in the appearance of local peak stresses [4-7]. According to [5], the peak stresses govern the resistance to plastic deformation to a significant degree. From an analysis of the experimental data [5-7], the deduction can be made that this stress microinhomogeneity, meaning also the resistance to plastic deformation, depends substantially on the loading and temperature modes. A rise in the loading rate and a reduction in the temperature result in an increase in the local peak stress fields, the appearance of significant elastic distortions of the crystal lattice. Such an increase in the microinhomogeneity results in an increase in the resistance to plastic deformation, as experiments show [4, 5].

However, the role of the peak stresses is not only to delay the development of plastic deformation. It follows from [6, 7] that the peak stresses exceeding the mean level are unstable and relax. This latter specifies numerous effects on the macrolevel: the relaxation of macrostresses, the delay in fluidity and creep, etc. The scalar measure, the temperature–time integral operator I, is taken as the microinhomogeneity characteristic of the stress state in a homogeneous continuous model of a solid. An approach to obtaining the operator I that is somewhat different from [2, 3] is proposed in this paper.

1. Let us represent an element of a polycrystalline body consisting of a large number of small particles in which the stresses are homogeneous and to which the mechanics of a continuous medium is applicable.

Let the stresses in particles at a specific time $t=s$ receive the increment

$$\Delta\sigma_{ij}(s) = \Delta\sigma^0_{ij}(s) + \Delta\sigma'_{ij}(s), \tag{1.1}$$

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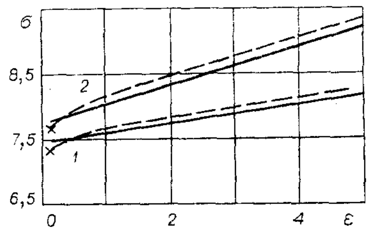

Fig. 1

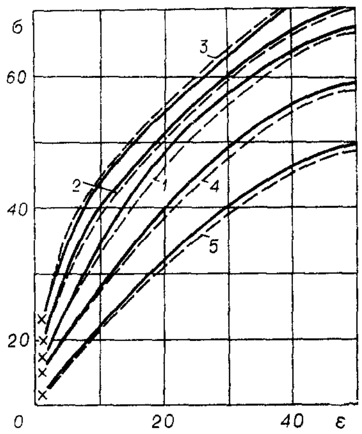

Fig. 2

where $\Delta\sigma^0_{ij}(s)$ is the increment in the mean level of the macrostresses, and $\Delta\sigma'_{ij}(s)$ is the deviation from the mean level.

The last term in (1.1) specifies the nonuniformity in the stress state. We give the increment $\Delta\sigma'_{ij}(s)$ after a certain time interval Δs in the form

$$\Delta\sigma'_{ij}(s) = A_{ijkl}\dot{\sigma}^0_{kl}\Delta s, \tag{1.2}$$

where $\dot{\sigma}^0_{kl}$ is the rate of change in the stress component at the macrolevel, and A_{ijkl} are random variables which change from particle to particle, and characterize the deviation of the stresses from the means in each. The fact that a change in some stress components implies a change in others is taken into account in (1.2).

On the basis of the capability of the peak stresses to relax, we take their change in time in the form [3]

$$d\left(\Delta\sigma'_{ij}(u)\right) = -\Delta\sigma'_{ij}(u)\,K(u-s)\,du, \tag{1.3}$$

where u is the running time $(s \leq u \leq t)$, $K(u-s)$ is a kernel that decreases with the passage of time and has the form

$$K(u-s) = \frac{\lambda}{2(\sqrt{a}+\sqrt{u-s})\sqrt{u-s}}, \tag{1.3a}$$

λ is the homological temperature, and a is a material constant.

Regular $\sqrt{a}+\sqrt{u-s}$ and singular $\sqrt{u-s}$ parts can be extracted in the denominator of (1.3a). The singular and regular parts separately describe the capacity of the microstresses to relax. The presence of the product of the singular and regular parts makes a kernel of the form (1.3a) sufficiently general and permits describing the influence of the temperature on the relaxation.

Integrating (1.3) with respect to u between s and t, we obtain

$$\Delta\sigma'_{ij}(t) = \Delta\sigma'_{ij}(s)\,Q(t,s),\; Q(t,s) = \exp\left[-\int_0^t K(t-s)\,ds\right]. \tag{1.4}$$

Taking account of (1.2) and (1.4), after integrating (1.1), we obtain the value of the stress at an arbitrary particle for an arbitrary time

$$\sigma_{ij}(t) = \sigma^0_{ij}(t) + A_{ijkl}\int_0^t \dot{\sigma}^0_{kl}(s)\,Q(t,s)\,ds. \tag{1.5}$$

It is assumed that all the numbers A_{ijkl} are independent and have the identical distribution function $P(A_{ijkl})$. Taking this into account, we substitute (1.5) into a known formula governing the elastic molding energy U_m in a body, and executing manipulations according to the method in [3], we obtain an expression for the mathematical expectation of the molding energy

$$\langle U_m\rangle = U^0_m + \frac{1}{9}A\left\{\left[\int_0^t(\dot{\sigma}^0_{xx}-\dot{\sigma}^0_{yy})\,Q(t,s)\,ds\right]^2 + \left[\int_0^t(\dot{\sigma}^0_{xx}-\dot{\sigma}^0_{zz})\,Q(t,s)\,ds\right]^2 + \left[\int_0^t(\dot{\sigma}^0_{yy}-\dot{\sigma}^0_{zz})\,Q(t,s)\,ds\right]^2 + \right.$$

$$+6\left[\int_0^t \dot{\tau}^0_{xy} Q(t,s)\,ds\right]^2 + 6\left[\int_0^t \dot{\tau}^0_{xz} Q(t,s)\,ds\right]^2 + 6\left[\int_0^t \dot{\tau}^0_{zy} Q(t,s)\,ds\right]^2\Bigg\}, \tag{1.6}$$

where U^0_m is the elastic energy in an ideal homogeneous body, A is the variance of the random variable A_{ijkl} to the accuracy of a factor, i.e.,

$$A = \frac{1}{6G}\int_{-\infty}^{\infty} (A_{ijkl})^2 P(A_{ijkl})\,dA_{ijkl}. \tag{1.7}$$

The second term in (1.6) is due to the nonuniformity in the stress distribution. It is also selected as a scalar quantity characterizing the microinhomogeneity of the stress state, i.e.,

$$I = \frac{1}{9}A\Bigg\{\left[\int_0^t (\dot{\sigma}^0_{xx} - \dot{\sigma}^0_{yy}) Q(t,s)\,ds\right]^2 + \left[\int_0^t (\dot{\sigma}^0_{xx} - \dot{\sigma}^0_{zz}) Q(t,s)\,ds\right]^2$$

$$+\left[\int_0^t (\dot{\sigma}^0_{yy} - \dot{\sigma}^0_{zz}) Q(t,s)\,ds\right]^2 + 6\left[\int_0^t \dot{\tau}^0_{xy} Q(t,s)\,ds\right]^2 + 6\left[\int_0^t \dot{\tau}^0_{xz} Q(t,s)\,ds\right]^2 + 6\left[\int_0^t \dot{\tau}^0_{zy} Q(t,s)\,ds\right]^2\Bigg\}, \tag{1.8}$$

where the variance A, defined according to (1.7), is later taken as a constant of the material, and only the constant A, and not A_{ijkl}, will enter into all the subsequent relationships.

The parameter I is inserted in the governing relationships of the theory of residual strain in terms of the resistance to plastic shear, whose assignment is basic in the slip concept. Dependences between the stress, strain, and time [2, 3] obtained here permit the description of a whole set of time effects at the microlevel under both simple and complex loadings.

2. When the stress and strain are considered in time, then the concept of proportional loading requires refinement. Let us introduce the concept of generalized proportional loading. In this case the stress components grow proportionally to some parameter at a constant rate (in the plastic domain).

Relationships between the stress, strain, and time [2, 3] are obtained mathematically complex for a generalized proportional loading. It is hence logical to propose simple dependences of the strain on the stress with time taken into account, on the basis of the theory of slip.

Let us take the proportionality of the deviators

$$\varepsilon_{ij} = (1/2)\gamma_i/\tau_i \cdot s_{ij}, \tag{2.1}$$

where s_{ij}, ε_{ij} are the stress and plastic strain deviator components, while τ_i and γ_i are the tangential stress and shear strain intensities. They are interconnected by a relationship independent of the kind of stress state:

$$\gamma_i = \gamma_i(\tau_i, I), \tag{2.2}$$

where $\gamma_i(\tau_i, I)$ is a monotonically increasing function in both arguments.

It is established experimentally that deviations from similarity of the deviators and deviations from the existence of a single strain curve [8, 9] are observed for proportional loading outside the fluidity limits. As is shown in [10], theories based on the slip concept describe these deviations. But they are insignificant, and hence, the dependences (2.2) and (2.1) will be used here. The function $\gamma_i(\tau_i, I)$ is selected so that under uniaxial tension the dependence of the stress on the strain, as determined by (2.1) and (2.2), and the dependence according to the slip concept [3] would agree. Consequently

$$\gamma_i(\tau_i, I) = \sqrt{3}k(1 - 1/\eta)^{3/2}(4\eta + 1),\ \eta = \frac{\sqrt{3}}{2}\frac{\tau_i}{F(\tau_i, I)},$$
$$F(\tau_i, I) = (1 - \lambda)\tau_0^*\left(1 + \frac{In}{\tau_i^2}\right), \tag{2.3}$$

where τ_0^* are constants, and $F(\tau_i, I)$ is a characteristic function of the material. The form of the function F agrees with that indicated in Sec. 1 above. Here τ_0^* is the shear yield point determined for low loading rates ($I \approx 0$). By adding the elastic to the plastic components in (2.1), we obtain the total strain.

There follows that $\tau_i = \tau_0$, $\gamma_i = 0$ at the proportionality limit and $\eta = 1$ follows from (2.3). We hence obtain an an equation to find the yield point τ_0

$$\frac{\sqrt{3}}{2}\tau_0 = (1-\lambda)\tau_0^*\left(1+\frac{I^n}{\tau_0^2}\right). \tag{2.4}$$

Let us examine a particular case of generalized proportional loading, uniaxial tension.

The parameter I defined by (1.8) is written in this case in the form

$$I = \frac{4a^2}{(1-\lambda)^2(2-\lambda)^2}\left\{(\dot{\sigma}_1^0)^2\left[\left(1+\sqrt{\frac{t}{a}}\right)^{2-\lambda} - \left(1+\sqrt{\frac{t-t_1}{a}}\right)^{2-\lambda} + (2-\lambda)\sqrt{\frac{t-t_1}{a}}\left(1+\sqrt{\frac{t-t_1}{a}}\right)^{1-\lambda} - (2-\lambda)\sqrt{\frac{t}{a}}\left(1+\sqrt{\frac{t}{a}}\right)^{1-\lambda}\right]^2 + (\dot{\sigma}_2^0)^2 \times\left[\left(1+\sqrt{\frac{t-t_1}{a}}\right)^{2-\lambda} - 1 - (2-\lambda)\sqrt{\frac{t-t_1}{a}}\left(1+\sqrt{\frac{t-t_1}{a}}\right)^{1-\lambda}\right]^2\right\}, \tag{2.5}$$

where t_1 is the time during which the material yield point is achieved, and $\dot{\sigma}^0$ is the rate of stress change that has one value before the yield point ($\dot{\sigma}^0 = \dot{\sigma}_1^0$). After it has been reached, the rate $\dot{\sigma}^0$ takes another value ($\dot{\sigma}^0 = \dot{\sigma}_2^0$).

Stress (σ, kg/mm^2) – relative elongation (ε, %) diagrams are constructed for different loading and temperature rates [solid lines in Figs. 1 and 2; the dashes are data from experiments in [11] for aluminum (Fig. 1), and tin bronze (Fig. 2)] by using (2.1)-(2.5). The diagrams are constructed for the following loading and temperature rates: aluminum, (curve 1) $\dot{\sigma}_2^0 = 2.3\cdot10^{-2}$ kg/mm$^2\cdot$sec, $\dot{\sigma}_2^0 = 4.3\cdot10^3$ kg/mm$^2\cdot$sec for $\lambda = 0.314$ (curve 2); tin bronze, $\dot{\sigma}_2^0 = 180$ kg/mm$^2\cdot$sec (curve 1), $\dot{\sigma}_2^0 = 900$ kg/mm$^2\cdot$sec (curve 2), $\dot{\sigma}_2^0 = 1800$ kg/mm$^2\cdot$sec for $\lambda = 0.226$ (curve 3), $\dot{\sigma}_2^0 = 180$ kg/mm$^2\cdot$sec for $\lambda = 0.376$ (curve 4), $\dot{\sigma}_2^0 = 180$ kg/mm$^2\cdot$sec for $\lambda = 0.455$ (curve 5).

It is seen from a comparison that the dependences proposed describe the experimentally observed increase in the yield point and degree of hardening with the rise in the loading rate and diminution in temperature satisfactorily (the yield point is denoted by a cross at each point). The following constants were taken in constructing the curves: for aluminum, G = 2700 kg/mm^2, n = 0.4, k = 0.014, $\tau_0^* = 3$ kg/mm^2 (G is the shear elastic modulus) and, for tin bronze, G = 2710 kg/mm^2, n = 0.49, k = 0.017, $\tau_0^* = 6$ kg/mm^2.

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